Powers and Exponents

Introduction

Powers and Exponents

- Repeated multiplication of the same number can be expressed in the form of exponents.
- Example: $625 = 5 \times 5 \times 5 \times 5$ *or* 5^4.
  Here '5' is the base raised to the power of 4, where 4 is the exponent and 5^4 is the exponential form of 625.

Powers with negative exponents

- Numbers can have positive powers which are called positive index. Example $a^n = a \times a \times a$.... n times.
- Numbers can also have negative powers such as $a^{-m} = \frac{1}{a^m} = \frac{1}{a \times a \times a \ldots\ldots\ldots\ldots m\ times}$
- Example : $5^{-3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125} = 0.008$

Visualising powers and exponents

- Example 1: 54 can be expressed as product of powers of prime numbers.
  $$54 = 2 \times 3 \times 3 \times 3 = 3^3 \times 2^1$$
- Example 2 :We know that $6^4 < 4^6$. This can be visualised as shown below:
  $$6^4 = 6 \times 6 \times 6 \times 6 = 1296$$
  $$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$$
  $$\therefore 6^4 < 4^6$$

Laws of Exponents

Powers with like bases

- $a^n \times a^m = a^{n+m}$.
  Example: $3^2 \times 3^4 = 3^6 = 729$
- $\frac{a^n}{a^m} = a^{n-m}$.
  Example: $2^5 \div 2^3 = \frac{32}{8} = 4 = 2^2$
- $a^m \times a^{-m} = a^m \times \frac{1}{a^m} = 1$

Power of a Power

- $(a^n)^m = a^{nm}$

Exponent Zero

- $a^m \times \frac{1}{a^m} = 1$
  $\Rightarrow \frac{a^m}{a^m} = a^{m-m} = a^0 = 1$

Powers with unlike bases and same exponent

- $a^n \times b^n = (ab)^n$
  Example: $2^2 \times 3^2 = 4 \times 9 = 36$ which is = $(2 \times 3)^2 = 6^2$
- $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
  Example: $\left(\frac{3^3}{4^3} = \frac{3}{4}\right)^3$
  $L.H.S.\ \frac{3^3}{4^3} = \frac{27}{64} = 0.42$
  $R.H.S.\ \left(\frac{3}{4}\right)^3 = 0.75^3 = 0.42$
  $\therefore\ L.H.S. = R.H.S.$

Uses of Exponents

Expanding a rational number using powers

- Rational Numbers can be expanded using exponents and powers.
- Example 1: 1284 can be written as $1 \times 10^3 + 2 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$.
- Example 2: 0.597 can be written as $5 \times 10^{-1} + 9 \times 10^{-2} + 7 \times 10^{-3}$.

Inter conversion between standard and normal forms

- Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**.
- Example:
  $43 = 4.3 \times 10 = 4.3 \times 10^1$
  $430 = 4.3 \times 100 = 4.3 \times 10^2$
  $4300 = 4.3 \times 1000 = 4.3 \times 10^3$
  $43000 = 4.3 \times 10000 = 4.3 \times 10^4$

Comparision of quantities using exponents

- If two numbers in standard form have the same power of 10, then the number with the larger factor is greater.
  E.g : $2.05 \times 10^3 > 1.05 \times 10^3$
- If two numbers in standard form have the same factor, then the number with the larger power of 10 will be greater.
  E.g $2.05 \times 10^6 > 2.05 \times 10^3$